Assignment 3

1a) Prove that F is a valid formula:

F = ƎxƎy¬P(a,x,f(g(y))) ∨ ƎxƎy(P(x,f(x),f(y)) Λ ¬Q(y,x)) ∨ ƎxQ(g(x),x)

¬F = ∀x∀yP(a,x,f(g(y))) Λ ∀v∀w(¬P(v,f(v),f(w)) ∨ Q(w,v)) Λ ∀z¬Q(g(z),z)

≡ ∀x∀y∀v∀w∀z(P(a,x,f(g(y))) Λ (¬P(v,f(v),f(w)) ∨ Q(w,v)) Λ ¬Q(g(z),z))

Clauses:

{{P(a,x,f(g(y)))}, {¬P(v,f(v),f(w)), Q(w,v)}, {¬Q(g(z),z)}} sub [w/g(z)] and [v/z] on clause 2

{{P(a,x,f(g(y)))}, {¬P(v,f(v),f(w)), Q(w,v)}, {¬Q(g(z),z)}, {¬P(z,f(z),f(g(z))), Q(g(z),z)}} use clauses 3 and 4

{{P(a,x,f(g(y)))}, {¬P(v,f(v),f(w)), Q(w,v)}, {¬Q(g(z),z)}, {¬P(z,f(z),f(g(z))), Q(g(z),z)}, {¬P(z,f(z),f(g(z)))}} sub [z/a] on clause 5

{{P(a,x,f(g(y)))}, {¬P(v,f(v),f(w)), Q(w,v)}, {¬Q(g(z),z)}, {¬P(z,f(z),f(g(z))), Q(g(z),z)}, {¬P(z,f(z),f(g(z)))}, {¬P(a,f(a),f(g(a)))}} sub [x/f(a)] and [y/a] on clause 1

{{P(a,x,f(g(y)))}, {¬P(v,f(v),f(w)), Q(w,v)}, {¬Q(g(z),z)}, {¬P(z,f(z),f(g(z))), Q(g(z),z)}, {¬P(z,f(z),f(g(z)))}, {¬P(a,f(a),f(g(a)))}, {P(a,f(a),f(g(a)))}} use clauses 6 and 7

{{P(a,x,f(g(y)))}, {¬P(v,f(v),f(w)), Q(w,v)}, {¬Q(g(z),z)}, {¬P(z,f(z),f(g(z))), Q(g(z),z)}, {¬P(z,f(z),f(g(z)))}, {¬P(a,f(a),f(g(a)))}, {P(a,f(a),f(g(a)))}, {□}}

Since empty clause is found, the negation of the formula is unsatisfiable, meaning the formula F is a valid formula.

1b) Check if logical consequence is true:

P1: ∀x(D(x) → H(x)) ≡ ∀x(¬D(x) ∨ H(x))

P2: ∀x∀y((I(x,y) Λ C(y)) → ¬Ǝz(I(x,z) Λ M(z))) ≡ ∀x∀y∀z((¬I(x,y) ∨ ¬C(y)) ∨ (¬I(x,z) ∨ ¬M(z)))

P3: ∀x(L(x) → ¬Ǝy(I(x,y) Λ H(y))) ≡ ∀x∀y(¬L(x) ∨ (¬I(x,y) ∨ ¬H(y)))

P4: Ǝx(I(c,x) Λ (C(x) ∨ D(x))) ≡ I(c,a) Λ (C(a) ∨ D(a))

C: L(c) → ¬Ǝz(I(c,z) Λ M(z)) ≡ ¬L(c) ∨ ∀z(¬I(c,z) ∨ ¬M(z))

¬C: L(c) Λ (I(c,a) Λ M(a))

F = P1 Λ P2 Λ P3 Λ P4 ⊨ C ≡ P1 Λ P2 Λ P3 Λ P4 Λ ¬C

Clauses:

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}} substitute [z/a] on clause 2

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}} use clauses 8 and 9

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}} substitute [x/c] on clause 10

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}, {¬I(c,y), ¬C(y), ¬I(c,a)}} use clauses 7 and 11

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}, {¬I(c,y), ¬C(y), ¬I(c,a)}, {¬I(c,y), ¬C(y)}} substitute [y/a] on clause 12

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}, {¬I(c,y), ¬C(y), ¬I(c,a)}, {¬I(c,y), ¬C(y)}, {¬I(c,a), ¬C(a)}} use clauses 7 and 13

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}, {¬I(c,y), ¬C(y), ¬I(c,a)}, {¬I(c,y), ¬C(y)}, {¬I(c,a), ¬C(a)}, {¬C(a)}} use clauses 5 and 14

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}, {¬I(c,y), ¬C(y), ¬I(c,a)}, {¬I(c,y), ¬C(y)}, {¬I(c,a), ¬C(a)}, {¬C(a)}, {D(a)}} substitute [x/a] on clause 1

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}, {¬I(c,y), ¬C(y), ¬I(c,a)}, {¬I(c,y), ¬C(y)}, {¬I(c,a), ¬C(a)}, {¬C(a)}, {D(a)}, {¬D(a), H(a)}} use clauses 15 and 16

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}, {¬I(c,y), ¬C(y), ¬I(c,a)}, {¬I(c,y), ¬C(y)}, {¬I(c,a), ¬C(a)}, {¬C(a)}, {D(a)}, {¬D(a), H(a)}, {H(a)}} substitute [y/a] on clause 3

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}, {¬I(c,y), ¬C(y), ¬I(c,a)}, {¬I(c,y), ¬C(y)}, {¬I(c,a), ¬C(a)}, {¬C(a)}, {D(a)}, {¬D(a), H(a)}, {H(a)}, {¬L(x), ¬I(x,a), ¬H(a)}} use clauses 17 and 18

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}, {¬I(c,y), ¬C(y), ¬I(c,a)}, {¬I(c,y), ¬C(y)}, {¬I(c,a), ¬C(a)}, {¬C(a)}, {D(a)}, {¬D(a), H(a)}, {H(a)}, {¬L(x), ¬I(x,a), ¬H(a)}, {¬L(x), ¬I(x,a)}} substitute [x/c] on clause 19

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}, {¬I(c,y), ¬C(y), ¬I(c,a)}, {¬I(c,y), ¬C(y)}, {¬I(c,a), ¬C(a)}, {¬C(a)}, {D(a)}, {¬D(a), H(a)}, {H(a)}, {¬L(x), ¬I(x,a), ¬H(a)}, {¬L(x), ¬I(x,a)}, {¬L(c), ¬I(c,a)}} use clauses 4 and 20

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}, {¬I(c,y), ¬C(y), ¬I(c,a)}, {¬I(c,y), ¬C(y)}, {¬I(c,a), ¬C(a)}, {¬C(a)}, {D(a)}, {¬D(a), H(a)}, {H(a)}, {¬L(x), ¬I(x,a), ¬H(a)}, {¬L(x), ¬I(x,a)}, {¬L(c), ¬I(c,a)}, {¬L(c)}} use clauses 6 and 21

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}, {¬I(c,y), ¬C(y), ¬I(c,a)}, {¬I(c,y), ¬C(y)}, {¬I(c,a), ¬C(a)}, {¬C(a)}, {D(a)}, {¬D(a), H(a)}, {H(a)}, {¬L(x), ¬I(x,a), ¬H(a)}, {¬L(x), ¬I(x,a)}, {¬L(c), ¬I(c,a)}, {¬L(c)}, {□}

Logical consequence is true since the formula f is unsatisfiable.

2a)

starter(green\_salad).

starter(melon).

starter(tomato\_salad).

starter(rabbit\_pate).

main(rock\_salmon).

main(roast\_beef).

main(pasta).

desert(cheese).

desert(yoghurt).

desert(montreal).

2b)

menu(hungry,X,Y,Z) :- starter(X), main(Y), desert(Z).

menu(not\_so\_hungry,X,Y) :- starter(X), main(Y).

menu(not\_so\_hungry,Y,Z) :- main(Y), desert(Z).

menu(on\_diet,X) :- starter(X).

2c)

sits\_left\_of(X,Y) :- sits\_right\_of(Y,X).

are\_neighbors\_of(X,Y,Z) :- sits\_left\_of(X,Z), sits\_right\_of(Y,Z).

next\_to\_each\_other(X,Y) :- sits\_right\_of(X,Y).

next\_to\_each\_other(X,Y) :- sits\_left\_of(X,Y).

3a)

path(X,Y) :- connected(X,Y).

path(X,Y) :- connected(X,Z), path(Z,Y).

3b)

move(X,Y) :- by\_Car(X,Y).

move(X,Y) :- by\_Train(X,Y).

move(X,Y) :- by\_Plane(X,Y).

travel(X,Y) :- move(X,Y).

travel(X,Y) :- move(X,Z), travel(Z,Y).